Notes

Twisted Beam Transducer: Frequency Shifts in Vibrating Modes of Beams under Twist

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A MINIATURE electromechanical device is described which measures input angles within a fraction of a second arc through conversion from a mechanical angular displacement to a difference in frequency, which is expressed as $\Delta\theta = B\Delta f$. The input angle $\Delta\theta$ is shown to be linearly dependent on the difference in transverse natural frequencies of two equally pretwisted flat tapes when their common-end junction is further twisted from its equilibrium pretwisted position θ_0 by the desired input angle. The tapes are identical in construction and are connected to each other at a common junction with their outer ends locked to a fixed reference at a predetermined pretwist angle θ_0. An input angle $\Delta\theta$ will twist one tape to a total angle $\theta_0 + \Delta\theta$ and untwist the second tape to an angle of $\theta_0 - \Delta\theta$. It is shown here that a change in twist angle changes the natural frequency of the tape and that the measurement of the difference in frequency between the two forementioned tapes, $f_1 - f_2$, is a direct measurement of the input angle $\Delta\theta$.

The input angle also may represent a linear displacement, such as the motion of a diaphragm, when translated to an angular motion. This displacement thus is converted directly to a difference-frequency similar to a frequency-modulated signal without distortion. This electromechanical transducer is miniature (about the size of a small matchbox), with a total power consumption in the order of 0.04×10^{-6}.

The derivation of the relation between input angle $\Delta\theta$ and transverse frequency difference $f_1 - f_2$, of two identical tapes twisted to angles $\theta_0 + \Delta\theta$ and $\theta_0 - \Delta\theta$, respectively, considers the total energy of the resonating system of a beam with its ends clamped. Each pretwisted tape is represented simply by a beam with varying moments of inertia under a constant axial tension, given by

$$I = I_1 + (I_2 - I_1)\sin^2\vartheta \tag{1}$$

where

I_1 = moment of inertia around (Z-Z) coordinate axis at zero pretwist ($\theta = 0°$)
I_2 = moment of inertia around (y-y) coordinate axis at zero pretwist ($\theta = 0°$)
ϑ = $\theta x/L$ = local twist angle of element dx along the beam length L
θ = total angle of tape twist

The wave shape of the vibrating beam clamped at both ends is assumed to be represented by $Yx = Y_0[1 + \cos(2\pi x/L)]e^{i\omega t}$. It was found in practice that ideal clamping conditions are not obtained to isolate total vibratory energy in the tape, and the error introduced by this uncertainty exceeds the error introduced by assuming the forementioned wave shape.

The natural frequency of a tape at zero twist is given by

$$f_0 = (1/1.015)\{(1/3mL)[T + (4\pi^2 EI_1/L^2)]\}^{1/2} \tag{2}$$

where T is axial tension, E modulus of elasticity, and m tape mass.

The natural frequency of a twisted tape is given by

$$f = f_0(1 + KA)^{1/2} = f_0\{1 + K[1 - (\sin\theta/\theta) + (\theta\sin\theta/(4\pi^2 - \theta^2))]\}^{1/2} \tag{3}$$

where

$$K = \frac{I_2 - I_1}{(L^2T/2\pi^2E) + 2I_1} \tag{4}$$

$$A = 1 - (\sin\theta/\theta) + [\theta\sin\theta/(4\pi^2 - \theta^2)] \tag{5}$$

The tape length L used to calculate Eq. (2) must be multiplied by a numerical constant higher than unity, for an equivalent added length, to compensate for the kinetic energy absorbed by the end clamps. This constant is obtained best experimentally.

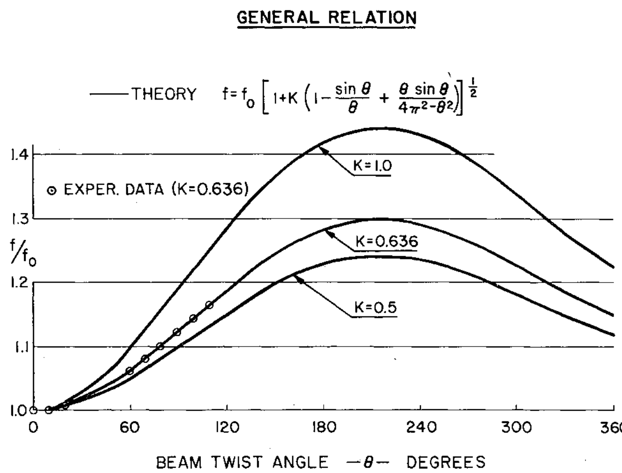

Fig. 1 Twisted beam theory

Expanding the transverse frequency solution for each tape in terms of a Taylor series, the transverse frequency difference of the two tapes is given by

$$f_1 - f_2 = 2(df/d\theta)\Delta\theta + \tfrac{1}{3}(d^3f/d\theta^3)\Delta\theta^3 + \ldots \tag{6}$$

This relation provides the calibration formula, since $f_1 - f_2$ is the direct quantity and $\Delta\theta$ is the indirectly measured input angle, where

$$\frac{df}{d\theta} = f_0(1 + KA)^{-1/2}\left(\frac{K}{2}\right)\left(\frac{dA}{d\theta}\right) \text{ cps/rad} \tag{7}$$

$$\frac{d^3f}{d\theta^3} = \frac{df}{d\theta}\left[\frac{d^3A}{d\theta^3} - \frac{3K}{2(1 + KA)}\left(\frac{d^2A}{d\theta^2}\right) + \frac{3K^2}{4(1 + KA)^2}\left(\frac{dA}{d\theta}\right)^2\right] \text{ cps/rad}^3 \tag{8}$$

The ratio of the frequencies of a single tape twisted to angle θ over its zero twist frequency is shown in Fig. 1.

Description of the Instrument

The sensing elements are shown in Fig. 2. The vibrating tapes also are shown assembled in Fig. 3 under item 1, fastened to the input arm 6 through diaphragm 4 and end clamps 2. The stationary end of the tape is clamped to the housing of

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the instrument 9 through a linear isolation spring assembly 5 that maintains constant tension during the operation. The spring assembly was removed during tests and was substituted by a hanging weight. Cross support tapes 7 and a tension wire 8 maintain stability of the rotating assembly with respect to the housing of the instrument while the assembly maintains free rotation.

The instrument has two tapes in series attached to each other through a spring couple 3. Rotation of the input arm 6 to the desired input angle will twist both tapes simultaneously. The tapes are pretwisted in opposite directions to each other to the desired pretwist angle before assembly. These metallic tapes are located in the transverse magnetic field between the poles of a pair of permanent magnets 11. A separate oscillator-amplifier is connected to the extreme ends of each tape which serves as a transducer of the oscillator-amplifier circuit. The tapes are energized by the output of the amplifier, which causes them to move in the magnetic field imposed by the permanent magnets. The motion of each tape induces a voltage across itself which is fed back to the oscillator-amplifier, and thus tape resonance is sustained. This resonant frequency is fed back to the oscillator-amplifier circuit, which may be measured with a frequency counter.

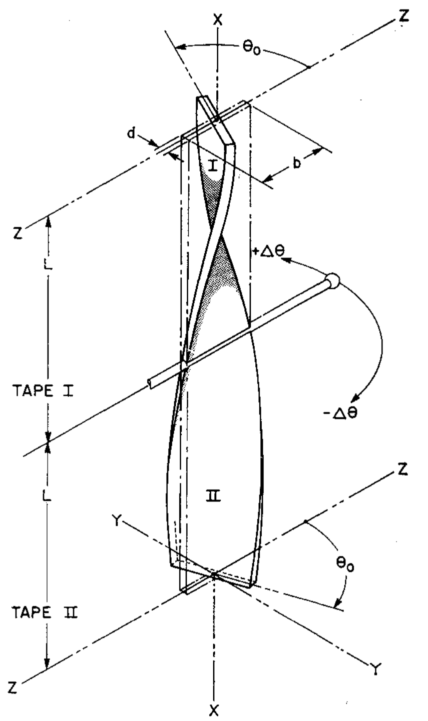

Fig. 2 Twisted beam elements

Experimental Verification

Experiments conducted from 0° to 360° of total twist angle confirm the relation of Eq. (3). Additional experiments were conducted to evaluate the sensitivity, linearity, and accuracy of Eq. (6). The total range of input angle tested was 45°, from 42.5° to 87.5° total twist on each tape, starting with a pretwist tape angle of 65°. The measured frequency difference of tapes having K equal to 1.06 is given by the following expression as a function of input angle:

$$f_1 - f_2 = 24.4984(1 - 0.000059\Delta\theta^2)\Delta\theta \qquad (9)$$

The maximum deviation from this relation during the entire range of input angle of 45° was within an error of 0.02 cps, equivalent to 3.2 sec arc. This error was within the accuracy of the input angle turntable on which the device was mounted, plus the error shift of the frequency counter.

An increase in width of the tiny tape by a factor of 2 will increase the sensitivity from 24.4984 just obtained to approximately 400, whereas the order of magnitude of the nonlinear term 0.000059 will not be affected. In most applications the nonlinear term may be ignored by using the following simplified expression:

$$f_1 - f_2 = (2df/d\theta)\Delta\theta \qquad (10)$$

The nonlinear term must be considered in applications for space guidance and navigational instrumentations when accuracies of a fraction of a second arc are desirable and the range of input angle is in the order of 45°.

Figure 1 shows that, theoretically, maximum frequency on one tape is reached at approximately 210° of total twist angle, but, in practice, absolute maximum frequency is reached at lower angles because, as mentioned earlier, part of the vibratory energy is transmitted to the end clamp assembly and the effective tape length is longer than the theoretical tape length, thus making the effective input angle larger. Experiments show that the ratio of the effective input angle to the actual input increases with K as a function of a third-order relation.

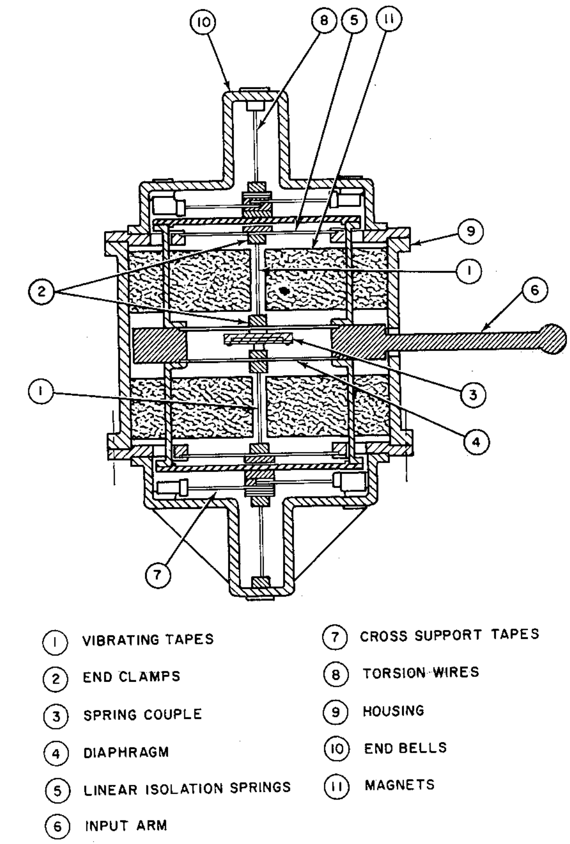

Fig. 3 Twisted beam transducer

Conclusions

The twisting beam transducer is a laboratory proven device that lends itself to a wide use in instrumentation, in space guidance components, and in telecommunications. Its inherent advantages are its ability to convert a mechanical motion directly to a difference in frequency output, its infinite threshold, which is limited only by the readout equipment, its high accuracy, and its miniature size. These characteristics present advantages for transmitting this frequency difference over long distances with repeated amplification and without signal distortion. The difference in frequency output signal is processed readily in a digital computer.

References

[1] High Command of the German Navy, "Arrangements for measuring velocity above ground of a ship or aircraft," German Patent 729894 (December 19, 1942).

[2] Voutsas, A. M., "Digital angle readout: a new physical concept of vibrating twisted beams," ARS Preprint 1632-61 (March 1961).

[3] Den Hartog, J., *Mechanical Vibrations* (McGraw-Hill Book Co., Inc., New York, 1940), 2nd ed., Chaps. II and IV.

Design Criteria for Wind-Induced Flight Loads on Large Boosted Vehicles

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Introduction

In rising through the atmosphere above the launch pad, a boosted vehicle may well experience sizable loads induced by rapidly changing wind conditions. To insure a high probability that the vehicle will traverse these winds without damage, adequate strength must be incorporated through the adoption of a realistic design philosophy and the development of design methods consistent with this philosophy. This paper is based upon a design philosophy that accepts a small percentage loss for any given vehicle when that vehicle is fired from a most critical geographical location during the worst season of the year. The loss rate considered herein is 1%.

In order to calculate the design load associated with a 1% loss rate, load statistics are derived directly by calculating the vehicle response from each individual sounding of a sample of wind soundings. This method, referred to as a "statistical load survey," is presented by Hobbs and his associates in Ref. 1.

Wing Sounding Samples

To determine design loads by means of a statistical load survey, an adequate sample is needed. Adequacy of the sample implies consistency with design philosophy, the greatest possible freedom from error and bias, and a sufficient size to provide both meaningful statistical information and a satisfactory representation of the climatology of any given launch site. To be consistent with the design philosophy specified previously, the sample should consist of measurements from the most severe geographical location during the season of the most severe winds (winter). The need for an error-free sample of winds is obvious, but its attainment is quite another matter. It generally is agreed that the accuracy and precision of present wind-measuring systems need considerable improvement.[2] For the present, one must be content using the best available data while urging the rapid development of improved measuring systems. Of the systems now available, only the AN/GMD-1 rawinsonde has been operational long enough to measure wind samples of the size needed for meaningful load statistics. Wind samples used in the present study are based on AN/GMD-1 measurements.

To represent the climatology of a given geographical location satisfactorily, a sample should include the variation in weather conditions from year to year. A 5- or 10-year period of data probably is representative of the general conditions at a given location, although there is no guarantee that more extreme conditions will not occur in the future.

In Ref. 1, the sample sizes associated with specific load accuracies and confidence levels are presented for a given vehicle-site combination. Typically, a sample size of 200 soundings usually provides a reasonable compromise, between the amount of calculation needed on the one hand, and the accuracy and confidence level for the determination of loads corresponding to a 1% loss rate on the other.

Using the foregoing criteria and sorting and reducing individual soundings by methods described in Ref. 3, wind samples of 200 soundings were made up for each of 11 geographical locations. Seven of these are distributed more or less evenly throughout the United States; they are Long Beach, Calif.; Denver, Colo.; Seattle, Wash.; Fort Worth, Tex.; International Falls, Minn.; Montgomery, Ala.; and Caribou, Me. Four foreign sites were considered also; they are Kadena, Okinawa; Tripoli, Libya; Bitburg, Germany; and Keflavik, Iceland. The Okinawa and Libya sites were chosen especially because of the occurrence of high winds in these areas. Keflavik was chosen so that the effects of winds in a cyclonic area could be studied.

Calculating Loads

Trajectories and loads were calculated by means of a five-degree-of-freedom rigid-body system of equations. For each vehicle considered, the most critical wind direction was chosen; for example, for a vehicle most sensitive to side winds, trajectories were flown normal to the mean wind direction. At representative stations on each vehicle, bending moments were calculated first in the pitch plane and yaw plane separately and then combined vectorially to obtain the resultant bending moment.

For any particular station on the vehicle, the design load for a given launch site is obtained by using the wind sample from that site, as follows. First, bending moment is calculated as a function of altitude for each of the 200 wind soundings in the sample. The peak bending moment from each sounding then is used to obtain a statistical distribution that defines the cumulative probability of exceeding a given bending moment.

Typical of the results of such a procedure are the data plotted in Fig. 1 which represent peak loads experienced

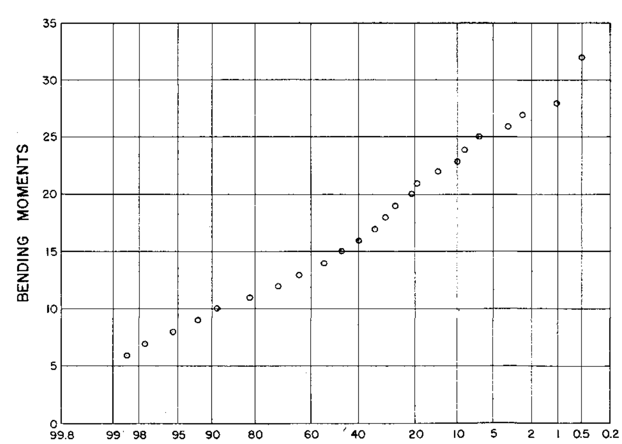

Fig. 1 Typical distribution of peak resultant bending moments

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